A Robust Test for Weak Instruments in Stata

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Abstract

We introduce and describe a Stata routine `weakivtest` implementing the test for weak instruments of Montiel Olea and Pflueger (2013). `weakivtest` allows for errors that are not conditionally homoskedastic and serially uncorrelated. It extends the Stock and Yogo (2005) weak instrument tests available in `ivreg2` and in the `ivregress` postestimation command `estat firststage`. `weakivtest` tests the null hypothesis that instruments are weak or that the estimator Nagar bias is large relative to a benchmark for both Two-Stage Least Squares (TSLS) and Limited Information Maximum Likelihood (LIML) with a single endogenous regressor. The routine can accommodate Eicker-Huber-White heteroskedasticity robust, Newey and West (1987) heteroskedasticity- and autocorrelation-consistent, and clustered variance estimates.

Keywords: F Statistic; Heteroskedasticity; Autocorrelation; Clustered; Stata

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1 Introduction

This paper describes and summarizes the weak instrument test of Montiel Olea and Pflueger (2013) and introduces a new Stata routine `weakivtest` implementing this test. `weakivtest` is a postestimation routine for `ivreg2` and `ivregress`.[2]

Weak instruments can bias point estimates and lead to substantial test size distortions (Nelson and Startz (1990); Stock and Yogo (2005)). Departures from the homoskedastic serially uncorrelated framework are not only extremely common in practice but can also further bias estimates and distort test sizes when instruments are weak (Montiel Olea and Pflueger (2013)). We provide a user-friendly routine for heteroskedasticity, autocorrelation, and clustering robust weak instrument tests. These tests apply to Two Stage Least Squares (TSLS) and Limited Information Maximum Likelihood (LIML) with one endogenous regressor.

Under strong instruments, TSLS and LIML are asymptotically unbiased. However, under weak instruments this is not the case. For overviews of the large literature on inference with potentially weak instruments see Stock et al. (2002) and Andrews and Stock (2006).

Staiger and Stock (1997) and Stock and Yogo (2005) proposed widely used pre tests for weak instruments under the assumption of conditionally homoskedastic, serially uncorrelated model errors. These tests reject the null hypothesis of weak instruments when the Cragg and Donald (1993) statistic exceeds a given threshold. This test statistic reduces to the first stage F statistic in the case with a single endogenous regressor. The null hypothesis of weak instruments can either be defined in terms of estimator bias or test size distortions.

The `ivreg2` suite, described in Baum et al. (2007) and Baum et al. (2010), implements the Stock and Yogo (2005) weak instrument test for the case of conditionally homoskedastic, serially uncorrelated model errors.

[2] While this paper provides a pre-test for weak instruments, methods for weak instrument robust inference are also available and are implemented in the command `weakiv` (Finlay et al. (2014)).

Practitioners frequently report the robust or non-robust first stage F statistic as an ad-hoc way of adjusting the Stock and Yogo (2005) tests for heteroskedasticity, autocorrelation, and clustering. However, Montiel Olea and Pflueger (2013) show that both the robust and the non-robust F statistics may be high even when instruments are weak. Baum et al. (2007) also emphasize that the Kleibergen and Paap (2006) rank Wald statistic does not provide a formal test for weak instruments in the presence of heteroskedastic, serially correlated, or clustered model errors.

`weakivtest` tests the null hypothesis that the estimator approximate asymptotic bias (or Nagar (1959) bias) exceeds a fraction τ of a "worst-case" benchmark. This benchmark agrees with the Ordinary Least Squares (OLS) bias when errors are conditionally homoskedastic and serially uncorrelated. The test rejects the null hypothesis when the test statistic, the effective F statistic, exceeds a critical value. The critical value depends on the significance level α, and the desired threshold τ.

When data is known to be conditionally homoskedastic and serially uncorrelated, the effective F statistic is identical to the Cragg and Donald (1993) statistic recommended by Stock and Yogo (2005). We can compare `weakivtest` critical values for the null hypothesis that the TSLS approximate asymptotic bias (henceforth, the Nagar bias) exceeds 10% of the benchmark to Stock and Yogo (2005) critical values for the null hypothesis that the TSLS bias exceeds 10% of the OLS bias. In the case with conditionally homoskedastic and serially uncorrelated errors, `weakivtest` critical values with significance level 5% increase from 8.53 for three instruments to 12.27 for 30 instruments. By comparison, the corresponding Stock and Yogo (2005) critical values increase from 9.08 for three instruments to 11.32 for 30 instruments.

2 Linear IV with Potentially Weak Instruments

We consider the following standard linear IV setup with one endogenous regressor and K instruments. We write the linear IV model in reduced form:

$$\mathbf{y} = \mathbf{Z\Pi}\beta + \mathbf{X}\boldsymbol{\gamma}_1 + \mathbf{v}_1, \tag{1}$$

$$\mathbf{Y} = \mathbf{Z\Pi} \quad + \mathbf{X}\boldsymbol{\gamma}_2 + \mathbf{v}_2. \tag{2}$$

Equation (1) denotes the reduced form second stage relationship. Equation (2) denotes the reduced form first stage relationship between the instruments and the endogenous regressor. The econometrician wishes to estimate the structural parameter β, while $\mathbf{\Pi} \in \mathbb{R}^K$ denotes the vector of unknown first stage parameters. $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ denote the vector of coefficients on the included exogenous regressors.

The econometrician observes the outcome variable $\mathbf{y}_s$, the endogenous regressor $\mathbf{Y}_s$, the vector of K instruments $\mathbf{Z}_s$ and the vector of L included exogenous regressors $\mathbf{X}_s$ for $s = 1, ..., S$. The unobserved reduced form errors have realizations $\mathbf{v}_{js}, j \in 1, 2$. We stack the realized variables in matrices $\mathbf{y} \in \mathbb{R}^S$, $\mathbf{Z} \in \mathbb{R}^{s\times K}$ and $\mathbf{v}_j \in \mathbb{R}^S$, $j \in \{1, 2\}$.

Two Stage Least Squares (TSLS) and Limited Information Maximum Likelihood (LIML) estimators depend on realized variables only through their projection residuals with respect to $\mathbf{X}$. Saving notation, from now on we let $\mathbf{y}$, $\mathbf{Y}$, and $\mathbf{v}_j$, $j = 1, 2$ denote their projection errors onto $\mathbf{X}$. For instance, we replace the endogenous regressor $\mathbf{Y}$ by $\mathbf{M_X Y}$, where $\mathbf{M_X} = \mathbb{I}_S - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. We also normalize the vector of instruments $\mathbf{Z}$ such that $\mathbf{Z}'\mathbf{Z}/S = \mathbb{I}_S$, which again leaves TSLS and LIML estimators unchanged.

Denote the projection matrix onto $\mathbf{Z}$ by $\mathbf{P_Z} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ and the complementary matrix by $\mathbf{M_Z} = \mathbb{I}_S - \mathbf{P_Z}$. The *Two Stage Least Squares* (TSLS) estimator of β is:

$$\widehat{\beta}_{TSLS} \equiv (\mathbf{Y}'\mathbf{P_Z}\mathbf{Y})^{-1}(\mathbf{Y}'\mathbf{P_Z}\mathbf{y}). \tag{3}$$

The *Limited Information Maximum Likelihood* (LIML) estimator of β is:

$$\widehat{\beta}_{LIML} = (\mathbf{Y}'(\mathbb{I}_S - k_{LIML}\mathbf{M_Z})\mathbf{Y})^{-1}(\mathbf{Y}'(\mathbb{I}_S - k_{LIML}\mathbf{M_Z})\mathbf{y}), \tag{4}$$

where k_{LIML} is the smallest root of the determinantal equation

$$\Big| [\mathbf{y}, \mathbf{Y}]'[\mathbf{y}, \mathbf{Y}] - k[\mathbf{y}, \mathbf{Y}]'\mathbf{M_Z}[\mathbf{y}, \mathbf{Y}] \Big| = 0. \tag{5}$$

The robust weak instrument pre test relies on two additional key assumptions. We model weak instruments by assuming that the IV first stage relation is local to zero, following the modeling strategy in Staiger and Stock (1997). Intuitively, the vector of first stage coefficients is small in magnitude relative to the sample size.

Assumption $\mathbf{L}_\Pi$. (**Local to Zero**) $\mathbf{\Pi} = \mathbf{\Pi}_S = \mathbf{C}/\sqrt{S}$, where $\mathbf{C}$ is a fixed vector $\mathbf{C} \in \mathbb{R}^K$.

We make high-level assumptions about the variances and covariances of the reduced form residuals and the residuals interacted with the vector of instruments.

Assumption HL. (**High Level**) The following limits hold as $S \to \infty$.

1. $\begin{pmatrix} \mathbf{Z}'\mathbf{v}_1/\sqrt{S} \\ \mathbf{Z}'\mathbf{v}_2/\sqrt{S} \end{pmatrix} \xrightarrow{d} \mathcal{N}_{2K}(\mathbf{0}, \mathbf{W})$ for some positive definite $\mathbf{W} = \begin{pmatrix} \mathbf{W}_1 & \mathbf{W}_{12} \\ \mathbf{W}_{12}' & \mathbf{W}_2 \end{pmatrix}$

2. $[\mathbf{v}_1, \mathbf{v}_2]'[\mathbf{v}_1, \mathbf{v}_2]/S \xrightarrow{p} \mathbf{\Omega}$ for some positive definite $\mathbf{\Omega} \equiv \begin{pmatrix} \omega_1^2 & \omega_{12} \\ \omega_{12} & \omega_2^2 \end{pmatrix}$

3. There exists a sequence of positive definite estimates $\{\widehat{\mathbf{W}}(S)\}$, measurable with respect to $\{y_s, Y_s, \mathbf{Z}_s\}_{s=1}^S$, such that $\widehat{\mathbf{W}}(S) \xrightarrow{p} \mathbf{W}$ as $S \to \infty$.

2.1 Testing Procedure

`weakivtest` tests the null hypothesis that instruments are weak. When the null hypothesis is rejected, the econometrician can conclude that instruments are strong and proceed using standard inference.

Montiel Olea and Pflueger (2013) use the standard Nagar (1959) methodology to obtain a tractable proxy for the asymptotic estimator bias. They define the Nagar bias as the expectation of the first three terms in the Taylor series expansion of the asymptotic estimator distribution under weak instrument asymptotics. The Nagar Bias is always defined and bounded for both TSLS and LIML. Montiel Olea and Pflueger (2013) define the null hypothesis of weak instruments such that the Nagar bias may be large. Under the alternative hypothesis, the Nagar bias is bounded relative to the benchmark.

Montiel Olea and Pflueger (2013) benchmark the TSLS Nagar bias N_{TSLS} and the LIML Nagar bias N_{LIML} against a function $BM(\beta, \mathbf{W}) \equiv \sqrt{tr(\mathbf{W}_1 - 2\beta\mathbf{W}_{12} + \beta^2\mathbf{W}_2)/tr(\mathbf{W}_2)}$. Intuitively, the benchmark BM captures the "worst-case" situation when instruments are completely uninformative and when first- and second-stage errors are perfectly correlated. It is also a natural extension of benchmarking against the Ordinary Least Squares bias when reduced form errors are conditionally homoskedastic, serially uncorrelated as in the tests proposed by Stock and Yogo (2005).

Under the weak instrument null hypothesis, the Nagar bias exceeds a fraction τ of the benchmark for at least some value of the structural parameter β and some direction of the first stage coefficients $\mathbf{\Pi}$. On the other hand, under the alternative, the Nagar bias is at most a fraction τ of the benchmark for any values for the structural parameter β and for any direction of the first stage coefficients $\mathbf{\Pi}$.

The robust weak instrument test rejects the null hypothesis of weak instruments when the test statistic, the *effective F statistic* $\widehat{F}_{eff}$

$$\widehat{F}_{eff} \equiv \frac{\mathbf{Y}'\mathbf{P_Z}\mathbf{Y}}{\mathrm{tr}(\widehat{\mathbf{W}}_2)}, \tag{6}$$

exceeds a critical value. In the just-identified case with one instrument, the effective F statistic equals the robust F statistic, but in general it differs from both the non-robust F

$$\widehat{F} \equiv \frac{\mathbf{Y}'\mathbf{P_Z}\mathbf{Y}}{K\widehat{\omega}_2^2} \tag{7}$$

and the robust F statistic

$$\widehat{F}_r \equiv \frac{\mathbf{Y}'\mathbf{Z}\widehat{\mathbf{W}}_2^{-1}\mathbf{Z}'\mathbf{Y}}{K \times S}. \tag{8}$$

The critical value c depends on the significance level α, the desired threshold τ, the estimated variance-covariance matrix $\widehat{W}$, and on the estimator (TSLS or LIML). Both a generalized and a simplified conservative critical value are available for TSLS.

3 Stata Implementation

1. `weakivtest` uses Stata's built-in `regress` routine to estimate (1) and (2) using equation-by-equation OLS. `weakivtest` estimates the matrix $\widehat{\mathbf{W}}$ using the same level of robustness as the preceding `ivreg2` or `ivregress` command with the user-written program `avar` of Baum and Schaffer (2013). The estimate $\widehat{\mathbf{W}}$ equals the robust estimated variance-covariance matrix times a degrees of freedom adjustment $\frac{S}{S-K-L-1}$. `weakivtest` supports estimating $\widehat{\mathbf{W}}$ with Eicker-Huber-White heteroskedasticity robust, Newey and West (1987) heteroskedasticity- and autocorrelation-consistent, or

clustered variance-covariance matrix estimates. $\widehat{\mathbf{\Omega}}$ is simply obtained as the cross product of $\widehat{\mathbf{v}}_1$ and $\widehat{\mathbf{v}}_2$.

2. `weakivtest` obtains the effective F statistic as a scaled version of the non-robust first stage F statistic $\hat{F}$ with $\widehat{F}_{eff} = \hat{F}\frac{K\hat{\omega}_2^2}{tr(\widehat{\mathbf{W}}_2)}$, where $\hat{\omega}_2$ is the consistent estimate of ω_2. We make three remarks:

   i) The asymptotic distribution of $\widehat{F}_{eff}$—denoted F^*_{eff}—is a weighted sum of non-central χ^2 random variables (see Montiel Olea and Pflueger (2013), Lemma 1, part 5, p. 362). One of the challenges in the implementation of our testing procedure is approximating the quantiles of such distribution.

   ii) Large values of the expectation of F^*_{eff} correspond to small values of the approximate asymptotic bias (or Nagar Bias) for both TSLS and LIML (see Montiel Olea and Pflueger (2013), Theorem 1, p. 362). This observation explains the selection of the test statistic.

3. `weakivtest` computes two key quantities that are used to approximate the upper α point of F^*_{eff}: a non-centrality parameter x that bounds the mean of F^*_{eff} under the null hypothesis and the effective degrees of freedom $\widehat{K}_{eff}$. The rationale for these parameters is as follows. Patnaik (1949) and Imhof (1961) approximate the critical values of a weighted sum of independent non-central χ^2 distributions by a central χ^2 with the same first and second moments. Building on this result, Montiel Olea and Pflueger (2013) approximate F^*_{eff} by the following non-central χ^2:

$$\frac{1}{\widehat{K}_{eff}}\chi^2_{\widehat{K}_{eff}}(\widehat{K}_{eff}x). \tag{9}$$

Here,

$$\widehat{K}_{eff} \equiv \frac{[\text{tr}(\widehat{\mathbf{W}}_2)]^2[1+2x]}{\text{tr}(\widehat{\mathbf{W}}_2'\widehat{\mathbf{W}}_2') + 2x\text{tr}(\widehat{\mathbf{W}}_2)\text{max eval}(\widehat{\mathbf{W}}_2)}, \tag{10}$$

$$x = B_e\left(\widehat{\mathbf{W}}, \widehat{\mathbf{\Omega}}\right)/\tau \text{ for } e \in \{TSLS, LIML\}, \tag{11}$$

and $B_e\left(\widehat{\mathbf{W}}, \widehat{\mathbf{\Omega}}\right)$ is closely related to the supremum of the Nagar bias relative to the benchmark.

Computing x requires maximizing the ratio of the Nagar (1959) bias divided by the benchmark over all values of the structural parameter β and all directions for the first stage coefficients $\mathbf{\Pi}$. As shown in Montiel Olea and Pflueger (2013), this step reduces to a numerical maximization over the real line and `weakivtest` implements it using the Stata built in function `optimize`.

4. Given the non-centrality parameter x and the effective degrees of freedom $\widehat{K}_{eff}$ the critical values can be calculated as the upper α point of $\chi^2_{\widehat{K}_{eff}}(x\widehat{K}_{eff})/\widehat{K}_{eff}$, following the curve fitting methodology of Patnaik (1949).

5. `weakivtest` calls routine `invnchi2` if Stata version is 13.1 or higher and calls routine `invnFtail` for lower Stata versions to compute the inverse non-central chi-squared CDF. At the time of writing, the built-in routine `invnchi2` does not support non-integer degrees of freedom for Stata version lower than 13.1. Setting the denominator degrees of freedom in `invnFtail` to a sufficiently large positive number approximates the non-central chi-squared CDF.

3.1 Syntax

`weakivtest` [, `level(#) eps(#) n2(#)`]

`level` specifies the confidence level. The default is level(0.05).

`eps` specifies the input parameter for the Nelder-Mead optimization technique. Its default value is set to 10^{-3}.

`n2` specifies the denominator degrees of freedom in `invnFtail`. The default value is 10^7.

The weak instrument test can adjust for a variety of violations of conditionally homoskedastic, independent, identically distributed model errors.

`weakivtest` supports the following `ivreg2` and `ivregress` options for the variance-covariance matrix:

1. `robust` estimates an Eicker-Huber-White heteroskedasticity robust variance-covariance matrix.

2. `cluster(`*varname*`)` estimates a variance-covariance matrix clustered by the specified variable.

3. `robust bw(#)` (for `ivreg2`) estimates a heteroskedasticity and autocorrelation-consistent variance-covariance matrix computed with a Bartlett (Newey-West) kernel with bandwidth #.

4. `bw(#)` without the `robust` option (for `ivreg2`) requests estimates that are autocorrelation-consistent but not heteroskedasticity-consistent.

5. `vce (hac nw #)` (for `ivregress`) estimates a heteroskedasticity and autocorrelation-consistent variance-covariance matrix computed with a Bartlett (Newey-West) kernel with number of lags #. The bandwidth of a kernel is equal to the number of lags plus one.

3.2 Remarks

`weakivtest` is a postestimation command for `ivreg2` and `ivregress`.

`weakivtest` reports the effective F statistic. It reports generalized TSLS and LIML critical values for threshold values $\tau \in \{5\%, 10\%, 20\%, 30\%\}$.

Montiel Olea and Pflueger (2013) provide both the generalized and a simplified conservative critical value for TSLS. The simplified critical value exploits an analytical conservative bound for the Nagar Bias of TSLS (see Montiel Olea and Pflueger (2013), Theorem 1, Part 3). The simplified procedure follows the same steps as the generalized procedure, but it sets the non-centrality parameter to $x = 1/\tau$. Hence, simplified critical values are computationally less demanding. For completeness, `weakivtest` saves both types of TSLS critical values. However, the TSLS generalized critical value provides a weakly more powerful test and should be used when available. `weakivtest` therefore displays only the TSLS generalized critical value.

4 Example

The example in this section implements `weakivtest` as a postestimation command for `ivreg2` using the data set of Campbell (2003) and Yogo (2004). The IV setup is identical to that in Table 2A of Montiel Olea and Pflueger (2013). This baseline example uses a Bartlett (Newey-West) kernel with bandwidth seven, a significance level of 5%, and focuses on a weak instrument threshold of $\tau = 10\%$.

By comparison, Montiel Olea and Pflueger (2013) report an effective F statistic of 7.94, a TSLS critical value of 15.49 and a LIML critical value of 9.68 for $\tau = 10\%$. `weakivtest` cannot reject the null of weak instruments for TSLS or for LIML for a weak instrument threshold of $\tau = 10\%$, consistent with the findings in Montiel Olea and Pflueger (2013).

```
. qui ivreg2 dc (rrf=z1 z2 z3 z4), robust bw(7)

. weakivtest
(obs=206)

Montiel-Pflueger robust weak instrument test
--------------------------------------------
Effective F statistic:        7.942
Confidence level alpha:          5%
--------------------------------------------

--------------------------------------------
Critical Values                TSLS      LIML
--------------------------------------------
% of Worst Case Bias
tau=5%                       25.848    15.245
tau=10%                      15.486     9.684
tau=20%                       9.817     6.569
tau=30%                       7.744     5.408
--------------------------------------------
```

5 Saved Results

`weakivtest` saves the following results in `r()`:

Scalars:

r(N)	Number of Observations
r(K)	Number of Instruments
r(n2)	Denominator degrees of freedom non-central F
r(level)	Test Significance Level
r(eps)	Optimization Parameter
r(F_eff)	Effective F Statistic
r(c_TSLS_5)	TSLS Critical Value for $\tau = 5\%$
r(c_TSLS_10)	TSLS Critical Value for $\tau = 10\%$
r(c_TSLS_20)	TSLS Critical Value for $\tau = 20\%$
r(c_TSLS_30)	TSLS Critical Value for $\tau = 30\%$
r(c_LIML_5)	LIML Critical Value for $\tau = 5\%$
r(c_LIML_10)	LIML Critical Value for $\tau = 10\%$
r(c_LIML_20)	LIML Critical Value for $\tau = 20\%$
r(c_LIML_30)	LIML Critical Value for $\tau = 30\%$
r(c_simp_5)	TSLS Simplified Conservative Critical Value for $\tau = 5\%$
r(c_simp_10)	TSLS Simplified Conservative Critical Value for $\tau = 10\%$
r(c_simp_20)	TSLS Simplified Conservative Critical Value for $\tau = 20\%$
r(c_simp_30)	TSLS Simplified Conservative Critical Value for $\tau = 30\%$

Scalars (continued):

`r(x_TSLS_5)`	TSLS Non-Centrality Parameter for $\tau = 5\%$
`r(x_TSLS_10)`	TSLS Non-Centrality Parameter for $\tau = 10\%$
`r(x_TSLS_20)`	TSLS Non-Centrality Parameter for $\tau = 20\%$
`r(x_TSLS_30)`	TSLS Non-Centrality Parameter for $\tau = 30\%$
`r(K_eff_TSLS_5)`	TSLS Effective Degrees of Freedom for $\tau = 5\%$
`r(K_eff_TSLS_10)`	TSLS Effective Degrees of Freedom for $\tau = 10\%$
`r(K_eff_TSLS_20)`	TSLS Effective Degrees of Freedom for $\tau = 20\%$
`r(K_eff_TSLS_30)`	TSLS Effective Degrees of Freedom for $\tau = 30\%$
`r(x_LIML_5)`	LIML Non-Centrality Parameter for $\tau = 5\%$
`r(x_LIML_10)`	LIML Non-Centrality Parameter for $\tau = 10\%$
`r(x_LIML_20)`	LIML Non-Centrality Parameter for $\tau = 20\%$
`r(x_LIML_30)`	LIML Non-Centrality Parameter for $\tau = 30\%$
`r(K_eff_LIML_5)`	LIML Effective Degrees of Freedom for $\tau = 5\%$
`r(K_eff_LIML_10)`	LIML Effective Degrees of Freedom for $\tau = 10\%$
`r(K_eff_LIML_20)`	LIML Effective Degrees of Freedom for $\tau = 20\%$
`r(K_eff_LIML_30)`	LIML Effective Degrees of Freedom for $\tau = 30\%$
`r(x_simp_5)`	TSLS Simplified Non-Centrality Parameter for $\tau = 5\%$
`r(x_simp_10)`	TSLS Simplified Non-Centrality Parameter for $\tau = 10\%$
`r(x_simp_20)`	TSLS Simplified Non-Centrality Parameter for $\tau = 20\%$
`r(x_simp_30)`	TSLS Simplified Non-Centrality Parameter for $\tau = 30\%$
`r(K_eff_simp_5)`	TSLS Simplified Effective Degrees of Freedom for $\tau = 5\%$
`r(K_eff_simp_10)`	TSLS Simplified Effective Degrees of Freedom for $\tau = 10\%$
`r(K_eff_simp_20)`	TSLS Simplified Effective Degrees of Freedom for $\tau = 20\%$
`r(K_eff_simp_30)`	TSLS Simplified Effective Degrees of Freedom for $\tau = 30\%$

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